Algebraic K-theory and Arithmetic
a conference in honour of J.Hurrelbrink
22-28 July 2012, Banach Center, Będlewo, Poland

ABSTRACTS

Speaker: Grzegorz Banaszak
Title: Brumer-Stark elements and K-theory of CM and totally real fields
Abstract: (joint paper with Cristian Popescu). I will discuss construction of special elements in K-groups of CM and totally real, abelian extensions of totally real fields. These special elements help to understand the structure of the group of divisible elements in even K-groups of the CM and totally real fields. The questions about structure of the group of divisible elements are directly related to well know conjectures in K-theory and algebraic number theory. Our construction of these special elements is based on a recent work of Greither and Popescu concerning Coates-Sinnott and Brumer-Stark conjectures.

Speaker: Spencer Bloch
Title: p-adic deformation of algebraic cycle classes
Abstract: (joint paper with H. Esnault and M. Kerz). Let $S = Spec\, W$ where W is the ring of Witt vectors over a perfect field. Let X/S be smooth and projective. Write X_n/S_n for the reduction modulo p^n. Using syntomic cohomology and the motivic cohomology of the closed fibre X_0, we construct a candidate for motivic cohomology for the thickenings X_n. We construct a chern character and use it to identify (tensor $\mathbb{Q}$) the continuous K_0 with motivic cohomology, assuming $p > dim X_0 + 6$. The naive inverse limit of $K_0(X_n)$ is a quotient of the continuous K_0.

Speaker: Mikhail Bondarko
Title: Coniveau filtrations and spectral sequences via weight structures
Abstract: Coniveau spectral sequences, coniveau filtrations, and Gersten resolutions relate the cohomology of a smooth variety X over a (perfect) field k with the one of its points. They are important for the study of several 'algebraic' cohomology theories (including K-theory, étale and singular cohomology). For a cohomology theory H^* given by a 'motivic complex' C (whose values are abelian groups) certain Gersten resolutions were constructed by Voevodsky. He deduced their existence from the following result: if $S/Spec\, k$ is a smooth semi-local scheme, S' is its generic point, then the morphisms $H^i(S) \to H^i(S')$ (for H as above) are split injective. His result also yield an expression of the coniveau spectral sequences corresponding to H in terms of the homotopy t-truncations of C in the category $DM_-^{eff}(k)$ (thus generalizing seminal results of Bloch and Ogus). The speaker extended these results to arbitrary cohomology theories that factorize through the triangulated category $DM_{gm}^{eff}(k)$ of Voevodsky's motives (one can also take any smooth primitive scheme for S in the split injectivity statement above). His main tool was the theory of weight structures (weight structures for triangulated categories are important cousins of t-structures, independently introduced by B. and by D. Pauksztello). He described a certain 'completion' of DM_{gm}^{eff} (the category of 'comotives'); if k is countable then comotives possess a weight structure whose heart consist of products of comotives for spectra of function fields over k. This immediately yields that coniveau spectral sequences could be defined for the cohomology of any motif $M \in Obj\, DM_{gm}^{eff}$; starting from E_2 they are DM_{gm}^{eff}-functorial (in M). The speaker also proved several 'splitting' results (for comotives and their cohomology).

Speaker: Jerzy Browkin
Title: K-groups of quaternion number fields
Abstract: I shall give some numerical examples of K-groups (i.e. class groups and tame kernels) of rings of integers of some number fields with the quaternion Galois group, and prove some results on their structure.

Speaker: Pierre Colmez
Title: Local-global compatibility in weight 1
Abstract: For modular forms of weight greater than 2, one can recover the smooth representation of GL2(Qp) attached to it using the locally algebraic vectors of the p-adic representation of GL2(Qp) obtained from the p-adic local Langlands correspondence. In weight 1, there are no locally algebraic vectors, but there is nevertheless a way to recover the smooth representation.

Speaker: Hélène Esnault
Title: Index of varieties over Henselian discrete valuation fields with algebraically closed residue field
Abstract: We show that if the index of a smooth projective variety over such a field divides the Euler characteristic of any coherent sheaf, and the class of the variety in the algebraic cobordism ring, if the residue characteristic is 0 or $p > 0$ larger than the dimension of the variety plus one, else it is true up to some p power. As a corollary, the index of a rationally connected variety over such a field is a p-power. (joint with Marc Levine and Olivier Wittenberg)

Speaker: Ivan Fesenko
Title: Higher geometric adeles and the Riemann-Roch theorem for 1-cycles on surfaces
Abstract: The Riemann-Roch theorem for proper curves over an arbitrary field can be easily deduced from adelic self-duality, this was already essentially included in 1950/51 Artin's Princeton lectures.

A higher adelic proof of the Rieman-Roch theorem for divisors on smooth irreducible projective surfaces over finite fields was recently announced by Osipov and Parshin. The proof relies, in addition to foundational papers by Beilinson and Parshin, on several more recent papers whose length is a three digital number of pages.

For further developments of adelic geometry, including an adelic interpretation of the intersection pairing, Arakelov intersection pairing and height pairing, concise proofs are needed. Following his stay in Moscow, the speaker found a several pages proof, relying just on the foundational papers, of a stronger property which implies the R-R theorem for irreducible proper surfaces over any field. This proof uses the full power of non-canonical self-duality of geometric adeles appropriately topologized. It combines adelic considerations with applications of the moving lemma, whose importance for adelic geometry was advocated by A. Beilinson. The paper is available from www.maths.nott.ac.uk/personal/ibf/ar.pdf

Speaker: Matthias Flach
Title: On the p-adic local invariant cycle theorem
Abstract: We report on joint work with Yitao Wu concerning a specialization map from the rigid cohomology of the geometric special fibre to D_{cris} of the p-adic etale cohomology of the geometric generic fibre of a flat, proper, generically smooth scheme over the ring of integers in a local field. We construct this map by descent from the semistable case and we show it is an isomorphism on the slope [0,1)-part if the scheme is regular. This uses a trace map on Witt vector cohomology for complete intersection morphisms recently constructed by Berthelot, Esnault and Ruelling.

Speaker: Thomas Geisser
Title: Rational motivic theories in characteristic p
Abstract: Parshin's conjecture states that higher K-theory of smooth and proper schemes over a finite field is torsion. In this talk, we discuss what this means for the four motivic theories (with rational coefficients) for arbitrary schemes over finite fields.

Speaker: Christian Haesemeyer
Title: Rational points and motivic homotopy theory
Abstract: We discuss recent joint work with A. Asok studying rational points on varieties "up to A^1 homotopy" and "up to stable A^1 homotopy". It turns out that the former is the same as an honest rational point, while the latter is the same as a zero cycle of degree one. We will describe the calculations of motivic (stable) homotopy groups needed to prove these results.

Speaker: Kevin Hutchinson
Title: The homology of the special linear group and pre-Bloch groups of fields.
Abstract: The pre-Bloch group (or scissors congruence group), P(F), of a field F is a group presented by generators and relations which derive from the five-term functional equation of the dilogarithm.

The Bloch group is a subgroup of $P(F)$ which, by a result of Suslin, is naturally a quotient of the indecomposable K_3 of F, and this in turn is a quotient of the group $H_3(SL(2,F),Z)$. Up to some possible 2-torsion, the kernel of the map $H_3(SL(2,F),Z) \to K_3^{ind}(F)$ coincides with the kernel of the stabilization map $H_3(SL(2,F),Z) \to H_3(SL(3,F),Z)$. We will describe how, for fields with valuations, lower bounds - and even exact computations - of this latter kernel can be expressed as direct sums of pre-Bloch groups of residue fields.

Speaker: Moritz Kerz
Title: Finiteness for Galois representation of function fields (after Deligne)
Abstract: Recently Deligne proved a finiteness theorem for the number of l-adic representations of the Galois group of a function field of arbitrary dimension over a finite field. He shows that for some fixed bounded ramification and fixed rank there are only finitely many Galois representations up to twist. In dimension one this is a direct consequence of Lafforgue's Langlands correspondence.

Speaker: Krzysztof Kłosin
Title: Congruences among Siegel and hermitian automorphic forms and bounds on Selmer groups
Abstract: We will present some recent results concerning the construction of congruences between automorphic forms on Sp(4) and U(2,2). The forms in question will be on one hand lifts from lower-rank groups (Yoshida lifts, Maass lifts) and on the other hand cusp forms on Sp(4) or U(2,2) with irreducible Galois representations. Since the Galois representations of the lifts are reducible we will discuss how these constructions imply the existence of non-trivial elements in the appropriate Selmer groups and hence one divisibility in the Bloch-Kato conjecture for the symmetric square L-function and the convolution L-function. Part of this is joint work with Mahesh Agarwal.

Speaker: Manfred Kolster
Title: K-Theory and Arithmetic — recent developments
Abstract: During his mathematical career Jürgen Hurrelbrink was always (but definitely not exclusively) interested in problems in K-Theory related to Algebraic Number Theory. We describe some of his work and recent related developments in the field, which we hope would have been of interest to him.

Speaker: Piotr Krasoń
Title: Linear relations in étale K-theory of curves
Abstract: (joint paper with Grzegorz Banaszak). We investigate linear independence over $\mathbb{Z}_l$ of elements in étale K-theory of a curve. This is done via reduction maps. The work is based on our previous result concerning linear independence over $\mathbb{Z}$ of elements in the Mordell-Weil group of an abelian variety defined over a number field.

Speaker: Aderemi Kuku
Title: Profinite Equivariant Higher Algebraic K-theory for the Action of Algebraic Groups
Abstract: Let G be an algebraic group over a field F. In this paper, we study profinite equivariant higher algebraic K-theory for the actions of G leading to computations of profinite higher algebraic K-theory of twisted flag varieties. More precisely, let ${}_\gamma\mathsf{F}$ be a twisted flag variety, B a finite dimensional separable F-algebra, and λ an odd rational prime. When F is a number field, we prove that for all $n \geqslant 1$ $\;K_{2n}^{pr}(({}_\gamma\mathsf{F}, B), \hat{Z}_\lambda))$ is an λ -complete Abelian group and $\operatorname{div} K_{2n}^{pr}(({}_\gamma\mathsf{F}, B), \hat{Z}_\lambda)) = 0$. If F is a p-adic field, we prove that for $n \geqslant 1$ $\;K_n^{pr}(({}_\gamma\mathsf{F}, B), \hat{Z}_\lambda)) \approx K_n(({}_\gamma\mathsf{F}, B), \hat{Z}_\lambda))$ are λ-complete profinite Abelian groups and $\operatorname{div} K_n^{pr}(({}_\gamma\mathsf{F}, B), \hat{Z}_\lambda)) = 0$.
As preliminary results, we prove that when F is a number field, then for the ordinary higher K-theories, we have $K_{2n+1}({}_\gamma\mathsf{F}, B)$ are finitely generated Abelian groups and $K_{2n}({}_\gamma\mathsf{F}, B)$ are torsion while if F is a p-adic field, then for all $n \geqslant 2$ $\;K_n({}_\gamma\mathsf{F}, B)_\lambda$ are finite groups.

Speaker: Jan Nekovář
Title: Congruence relations and cohomology of quaternionic Shimura varieties
Abstract: We are going to explain what information about etale cohomology of quaternionic Shimura varieties can be obtained from generalised Eichler-Shimura relations, without counting points over finite fields

Speaker: Raman Parimala
Title: Splitting the ramification of algebras over arithmetic surfaces.
Abstract: Saltman's work on the splitting of ramification of elements in a Brauer group of function fields of surfaces has been very effectively used in the work of Parimala-Suresh in determining the u-invariant of function fields of p-adic curves. We discuss the corresponding question for function fields of curves over number fields. Splitting the ramification of algebras leads to questions on period-index for algebras over arithmetic surfaces which have an affirmative answer granting a conjecture of Colliot Thelene on a local global principle for existence of zero cycles of degree one, which is wide open.

Speaker: Popescu Cristian
Title: Special values of L-functions and Quillen K-groups
Abstract: We will discuss our joint work with Greither on the Brumer-Stark and Coates-Sinnott conjectures, linking special values of equivariant Artin L-functions to the Galois module structure of even Quillen K-groups of rings of integers of global fields.

Speaker: Hourong Qin
Title: Anomalous primes and quadratic polynomials capture their primes
Abstract: Let $D \in \mathbb{Z}$ be an integer which is neither a square nor a cube in $\mathbb{Q}(\sqrt{-3})$ and define $E_D : y^2 = x^3 + D$. B. Mazur conjectured that there are infinitely many anomalous primes for the elliptic curve E_D. We show that the Hardy-Littlewood Conjecture implies the Mazur's conjecture.

Speaker: Ulf Rehmann
Title: On the mathematical work of Jurgen Hurrelbrink
Abstract: A survey on various aspects of Jurgen Hurrelbrink's mathematical work will be given, covering in particular his achievements in the areas of arithmetic groups and quadratic forms.

Speaker: Peter Stevenhagen
Title: Unit signatures in real quadratic fields
Abstract: We will be concerned with counting the number of real quadratic number fields for which the fundamental unit has norm -1. In elementary terms, this amounts to counting the number of positive squarefree integers d for which the negative Pell equation

$$x^2 + dy^2 = -1$$

admits a solution in integers.

I will present an old conjecture of mine that provides a very precise answer: among the real quadratic fields containing elements of norm -1, a fraction

$$P = 1 - \prod_{j \geqslant 1 \text{ odd}} (1 - 2^{-j}) = .5805775582....$$

has a fundamental unit of norm -1. The conjecture is based on a plausible but unproved equidistribution result on Frobenius elements.

Recent progress of Fouvry and Klüeners (Annals 2010) in analytic number theory has now given rise to partial results in the direction of the conjecture. The full conjecture remains wide open to date.

Speaker: Takeshi Tsuji
Title: p-adic perverse sheaves and arithmetic D-modules with singularities along a simple normal crossing divisor
Abstract: I will discuss a generalizations of the theory of crystalline p-adic representations/sheaves and filtered Frobenius modules by G. Faltings and O. Brinon in the relative case, to p-adic perverse sheaves for the stratification given by a simple normal crossing divisor. We use log etale cohomology and log arithmetic D-modules.

Speaker: Jerzy Urbanowicz
Title: On some new congruences of Emma Lehmer's type
Abstract: Let $s \in \{1,2,3\}$, $r|24$, $r > 1$ and $k \geqslant 1$. Assume that $n > r$ is an odd natural number, r is relatively prime to n and χ_n denotes the trivial character modulo n.

We are going to talk on our recent results obtained in two papers [5], resp. [6]. The papers are on congruences for the sums $T_{r,k}(n) = \sum_{i=1}^{[n/r]} \frac{\chi_n(i)}{i^k} \pmod{n^s}$, resp. $U_r(n) = \sum_{i=1}^{[n/r]} \frac{\chi_n(i)}{n-ri} \pmod{n^s}$.

The congruences obtained in [5] are consequences of an identity proved in [10], which was earlier successfully exploited to solve some other problems. The congruences generalize those obtained in [8], [7] and [9] in the case when $n = p$ is an odd prime. We obtain 82 new congruences for $T_{r,k}(n) \pmod{n^s}$. Two congruences for $T_{r,k}(n) \pmod{n^2}$ were proved in [1], resp. [4] for $(r,k) = (2,1)$, resp. $(4,2)$.

The paper [6] is an appendix to the paper [5]. In [6] we deduced some new congruences for the sums $U_r(n) \pmod{n^s}$ using those for $T_{r,k}(n)$ proved in [5]. Our congruences for $s = 2$ extend those published in [2] and [3] for $r \in \{2,3,4,6\}$ and n odd, not divisible by 3. These four congruences have the same form as those proved by E. Lehmer in her paper [7] in the case when $n = p$ is an odd prime. The sums $U_r(n) \pmod{n^2}$ are rational linear combinations of Euler's quotients. In the case when $r \in \{8,12,24\}$, omitted in [7], [2] and [3], the sums $U_r(n) \pmod{n^2}$ are linear combinations of the Euler quotients and three generalized Bernoulli numbers $\frac{1}{n\phi(n)} B_{n\phi(n),\chi} \prod_{p|n} \left(1 - p^{n\phi(n)-1}\right)$ attached to even quadratic characters χ of conductor dividing 24. Also some new congruences for $s = 3$ with one additional summand $-\frac{n^2}{2r^3} B_{n^2\phi(n)-2} \prod_{p|n} \left(1 - p^{n^2\phi(n)-3}\right)$ for all $r|24$ are obtained.

References

[1] T.X. Cai, A congruence involving the quotients of Euler and its applications (I), *Acta Arithmetica* **1**03 (2002), 313–320.
[2] T.X. Cai, X.D. Fu, X. Zhou, A congruence involving the quotients of Euler and its applications (II), *Acta Arithmetica* **1**30 (2007), 203–214.
[3] H.-Q. Cao, H. Pan, Note on some congruences of Lehmer, *J. Number Theory* **1**29 (2009), 1813–1819.
[4] S. Kanemitsu, J. Urbanowicz, N.-L. Wang, *On some new congruences for generalized Bernoulli numbers*, accepted for publication in Acta Arithmetica, 2012.
[5] S. Kanemitsu, T. Kuzumaki, J. Urbanowicz, *On congruences for the sums* $\sum_{i=1}^{[n/r]} \frac{\chi_n(i)}{i^k}$ *of E. Lehmer's type*, submitted for publication, 2012.
[6] T. Kuzumaki, J. Urbanowicz, *On congruences for the sums* $\sum_{i=1}^{[n/r]} \frac{\chi_n(i)}{n-ri}$ *of E. Lehmer's type*, submitted for publication, 2012.
[7] E. Lehmer, On congruences involving Bernoulli numbers and the quotients of Fermat and Wilson, *Annals of Math.* **3**9 (1938), 350–360.
[8] M. Lerch, Zur Theorie des Fermatschen Quotienten $\frac{a^{p-1}-1}{p} = q(a)$, *Math. Annalen* **6**0 (1905), 471–490.
[9] Z.-H. Sun, Congruences involving Bernoulli and Euler numbers, *J. Number Theory* **1**28 (2008), 280–312.
[10] J. Szmidt, J. Urbanowicz and D. Zagier, Congruences among generalized Bernoulli numbers, *Acta Arithmetica* **7**1 (1995), 273–278.

Speaker: Charles Weibel
Title: Unstable operations in etale and motivic cohomology
Abstract: In this talk we analyze the many motivic cohomology operations which have been constructed, and attempt to classify them. This is joint work with Bert Guillou. We first classify all etale cohomology operations. This gives new motivic cohomology operations in the Beilinson-Lichtenbaum range and allows us to construct previously unknown motivic operations. For example, multiplication by a torsion element in the Brauer group is an operation from $H^{1,2}(X)$ to $H^{3,3}(X)$. We then classify all motivic operations on $H^{n,1}$.

Speaker: Jörg Wildeshaus
Title: Intermediate extensions of Chow motives of Abelian type
Abstract: A profound conjecture concerning the category of motives over some base S predicts the existence of a t-structure, all of whose realizations are compatible with the so-called *perverse* t-structure. This structure would in particular allow for the construction of the *intermediate extension* to S of any Chow motive over an open sub-scheme U of S, canonically up to (unipotent) automorphisms restricting to the identity on U.

The aim of the talk is to sketch an unconditional alternative to the approach *via* t-structures, for Chow motives "of Abelian type" (more generally, "with Abelian degeneration to $S - U$"), under certain geometric conditions on S and U. This alternative is based on two important notions: *weight structures* à la Bondarko, and *semi-primary categories* à la André–Kahn. The geometric conditions in question turn out to be satisfied for example if U is a (smooth) Shimura variety, and S its Baily–Borel compactification.